Test case 3a.9: Pure yaw

Conditions

- Captive pure yaw motion in still water
- Fixed (Even keel)
- Without rudders
- With bilge keels

Pre scribed PMM motions:

- Sway motion: $\eta_{PMM} = -2S_{mm}\sin\left(\frac{2\pi N}{60}t\right)$
- Sway velocity: $v_{PMM} = -2\left(\frac{2\pi N}{60}\right)S_{mm}\cos\left(\frac{2\pi N}{60}t\right)$
- Sway acceleration: $\dot{v}_{PMM} = 2\left(\frac{2\pi N}{60}\right)^2 S_{mm}\sin\left(\frac{2\pi N}{60}t\right)$
- Heading angle: $\psi = -\psi_0\cos\left(\frac{2\pi N}{60}t\right)$
- Yaw rate: $r = r_{PMM} = \psi_0\left(\frac{2\pi N}{60}\right)\sin\left(\frac{2\pi N}{60}t\right)$
- Yaw acceleration: $\dot{r} = \dot{r}_{PMM} = \psi_0\left(\frac{2\pi N}{60}\right)^2\cos\left(\frac{2\pi N}{60}t\right)$

F_n [-]	R_n [-]	U_C [m/s]	N [rpm]	S_{mm} [m]	ψ_0 [deg]	r'_{max} [-]
0.280	4.643×10^6	**1.531**	8.0210	0.1636	10.2	0.30

Items and Remarks

Figure Number	Items	Remarks
Fig. 3a.9-1	Time history of non-dimensionalized longitudinal force (X')	To be compared with experimental results download
Fig. 3a.9-2	Time history of non-dimensionalized transverse force (Y')	To be compared with experimental results download
Fig. 3a.9-3	Time history of non-dimensionalized yaw moment (N')	To be compared with experimental results download
Fig. 3a.9-4	Damping parts of Y' and N' vs. r'	To be compared with experimental results download
Fig. 3a.9-5	Inertial parts of Y' and N' vs. $\dot{r}'$	To be compared with experimental results download

- Coordinate system for comparision is ship-fixed at midship on the undisturbed waterplane.

- $F_n = \dfrac{U_C}{\sqrt{gL_{PP}}}$, $R_n = \dfrac{U_C \cdot L_{PP}}{\nu}$

  where, U_C is towing carriage speed, g is the gravitational acceleration and ν is the kinematic viscosity of water.

- All quantities are non-dimensionalized with water density (ρ), ship speed ($U = \sqrt{u^2 + v^2}$), lateral underwater area ($A_0 = L_{pp} T_m$), and the length between perpendiculars (L_{pp}).

$$\mathrm{X}' = \frac{\mathrm{F}_{\mathrm{X}_{\mathrm{Hydro}}}}{0.5\rho \mathrm{U}^2 A_0} = \frac{F_{X_{total}} + M(\dot{u} - rv - X_G r^2 - Y_G \dot{r})}{0.5\rho \mathrm{U}^2 T_m L_{pp}}$$

$$\mathrm{Y}' = \frac{\mathrm{F}_{\mathrm{Y}_{\mathrm{Hydro}}}}{0.5\rho \mathrm{U}^2 A_0} = \frac{F_{Y_{total}} + M(\dot{v} + ru - Y_G r^2 + X_G \dot{r})}{0.5\rho \mathrm{U}^2 T_m L_{pp}}$$

$$\mathrm{N}' = \frac{\mathrm{M}_{\mathrm{Z}_{\mathrm{Hydro}}}}{0.5\rho \mathrm{U}^2 A_0 \mathrm{L}_{\mathrm{pp}}} = \frac{M_{Z_{total}} + I_Z \dot{r} + M(X_G(\dot{v} + ru) - Y_G(\dot{u} - rv))}{0.5\rho \mathrm{U}^2 T_m \mathrm{L}_{\mathrm{pp}}^2}$$

- From the Fourier series expansion of Y' and N',

$$Y' = a_o + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t)$$
$$N' = c_o + \sum_{n=1}^{\infty} c_n \cos(n\omega t) + \sum_{n=1}^{\infty} d_n \sin(n\omega t)$$

damping parts are

$$Y'_D = \sum_{n=1}^{3} b_n \sin(n\omega t)$$
$$N'_D = \sum_{n=1}^{3} d_n \sin(n\omega t)$$

and inertial parts are

$$Y'_T = a_1 \cos(\omega t)$$
$$N'_T = c_1 \cos(\omega t)$$

where,

$$\omega = \frac{2\pi N}{60}$$

- Non-dimensional yaw rate and accelerations:

$$r' = \frac{r L_{PP}}{U_C}$$
$$\dot{r}' = \dot{r}\left(\frac{L_{PP}}{U_C^2}\right)^2$$