

## Test case 3a.12: Pure yaw



### Conditions

- Captive pure yaw motion in still water
- Pitch, heave, and roll free
- Without rudders
- With bilge keels

*Pre scribed PMM motions:*

- Sway motion:  $\eta_{PMM} = -2S_{mm} \sin\left(\frac{2\pi N}{60} t\right)$
- Sway velocity:  $v_{PMM} = -2\left(\frac{2\pi N}{60}\right) S_{mm} \cos\left(\frac{2\pi N}{60} t\right)$
- Sway acceleration:  $\dot{v}_{PMM} = 2\left(\frac{2\pi N}{60}\right)^2 S_{mm} \sin\left(\frac{2\pi N}{60} t\right)$
- Heading angle:  $\psi = -\psi_0 \cos\left(\frac{2\pi N}{60} t\right)$
- Yaw rate:  $r = r_{PMM} = \psi_0 \left(\frac{2\pi N}{60}\right) \sin\left(\frac{2\pi N}{60} t\right)$
- Yaw acceleration:  $\dot{r} = \dot{r}_{PMM} = \psi_0 \left(\frac{2\pi N}{60}\right)^2 \cos\left(\frac{2\pi N}{60} t\right)$

$F_n$ [-]	$R_n$ [-]	$U_C$ [m/s]	$N$ [rpm]	$S_{mm}$ [m]	$\psi_0$ [deg]	$r'_{\max}$ [-]
0.280	$4.643 \times 10^6$	<b>1.531</b>	<b>8.0210</b>	<b>0.1636</b>	<b>10.2</b>	0.30

## Items and Remarks

Figure Number	Items	Remarks
Fig. 3a.12-1	Time history of non-dimensionalized longitudinal force ( $X'$ )	To be compared with experimental results <a href="#">download</a>
Fig. 3a.12-2	Time history of non-dimensionalized transverse force ( $Y'$ )	To be compared with experimental results <a href="#">download</a>
Fig. 3a.12-3	Time history of non-dimensionalized yaw moment ( $N'$ )	To be compared with experimental results <a href="#">download</a>
Fig. 3a.12-4	Time history of pitch angle	To be compared with experimental results <a href="#">download</a>
Fig. 3a.12-5	Time history of heave motion	To be compared with experimental results <a href="#">download</a>
Fig. 3a.12-6	Time history of roll motion	To be compared with experimental results <a href="#">download</a>
Fig. 3a.12-7	Damping parts of $Y'$ and $N'$ vs. $r'$	To be compared with experimental results <a href="#">download</a>
Fig. 3a.12-8	Inertial parts of $Y'$ and $N'$ vs. $\dot{r}'$	To be compared with experimental results <a href="#">download</a>

- Coordinate system for comparison is ship-fixed at midship on the undisturbed waterplane.

$$\bullet \quad F_n = \frac{U_c}{\sqrt{gL_{PP}}}, \quad R_n = \frac{U_c \cdot L_{PP}}{\nu}$$

where,  $U_c$  is towing carriage speed,  $g$  is the gravitational acceleration and  $\nu$  is the kinematic viscosity of water.

- All quantities are non-dimensionalized with water density ( $\rho$ ), ship speed ( $U = \sqrt{u^2 + v^2}$ ), lateral underwater area ( $A_0 = L_{pp} T_m$ ), and the length between perpendiculars ( $L_{pp}$ ).

$$X' = \frac{F_{X_{Hydro}}}{0.5\rho U^2 A_0} = \frac{F_{X_{total}} + M(\dot{u} - rv - X_G r^2 - Y_G \dot{r})}{0.5\rho U^2 T_m L_{pp}}$$

$$Y' = \frac{F_{Y_{\text{Hydro}}}}{0.5\rho U^2 A_0} = \frac{F_{Y_{\text{total}}} + M(\dot{v} + ru - Y_G r^2 + X_G \dot{r})}{0.5\rho U^2 T_m L_{pp}}$$

$$N' = \frac{M_{Z_{\text{Hydro}}}}{0.5\rho U^2 A_0 L_{pp}} = \frac{M_{Z_{\text{total}}} + I_Z \dot{r} + M(X_G(\dot{v} + ru) - Y_G(\dot{u} - rv))}{0.5\rho U^2 T_m L_{pp}^2}$$

- From the Fourier series expansion of  $Y'$  and  $N'$ ,

$$Y' = a_o + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t)$$

$$N' = c_o + \sum_{n=1}^{\infty} c_n \cos(n\omega t) + \sum_{n=1}^{\infty} d_n \sin(n\omega t)$$

damping parts are

$$Y'_D = \sum_{n=1}^3 b_n \sin(n\omega t)$$

$$N'_D = \sum_{n=1}^3 d_n \sin(n\omega t)$$

and inertial parts are

$$Y'_T = a_1 \cos(\omega t)$$

$$N'_T = c_1 \cos(\omega t)$$

where,

$$\omega = \frac{2\pi N}{60}$$

- Non-dimensional yaw rate and accelerations:

$$r' = \frac{r L_{PP}}{U_C}$$

$$\dot{r}' = \dot{r} \left( \frac{L_{PP}}{U_C^2} \right)^2$$